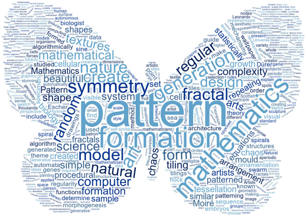
Advanced Mathematical Perspectives 1

Lecture 6: Irregular tilings



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Section 1

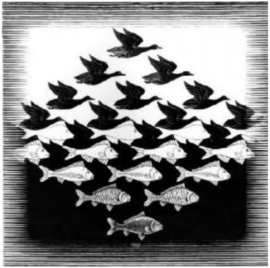
Symmetry Breaking and Aperiodic Tessellations

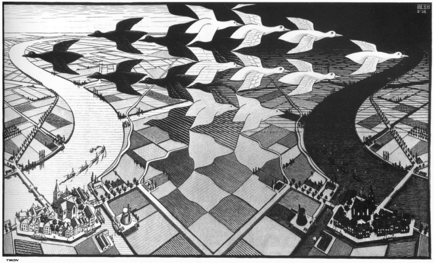
Broken symmetry

- Lots of superficial symmetries
  - in nature, *e.g.*, bilateral
  - in architecture, *e.g.*, many buildings
  - in design, *e.g.*, a knife
- But equally often, they are not exactly symmetric
  - where is your heart?
  - external view of a building may hide interior asymmetry
  - left and right-handed scissors
- Where does this asymmetry come from (in nature)?
  - lots of (complicated) answers, but for the moment, let's just look at asymmetric tilings

Symmetry in Art

- Strict symmetries have often been used in ornamentation, but sometimes seem to be considered too simple to be "art"
- Many of Escher's more powerful works are based around tessellations but somehow broken





Escher's "Sky and Water" and "Day and Night"

- How an artist breaks away from underlying patterns is often a key to their impact.

Aperiodic tilings

- All of the tilings we considered are *periodic*
  - the pattern repeats
  - they have translation symmetries
- What if we loosen that criteria? Are there any *simple* tilings that are aperiodic?

Aperiodic tilings

- All of the tilings we considered are *periodic*
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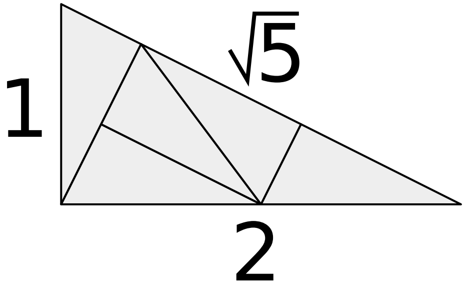
- Yes!

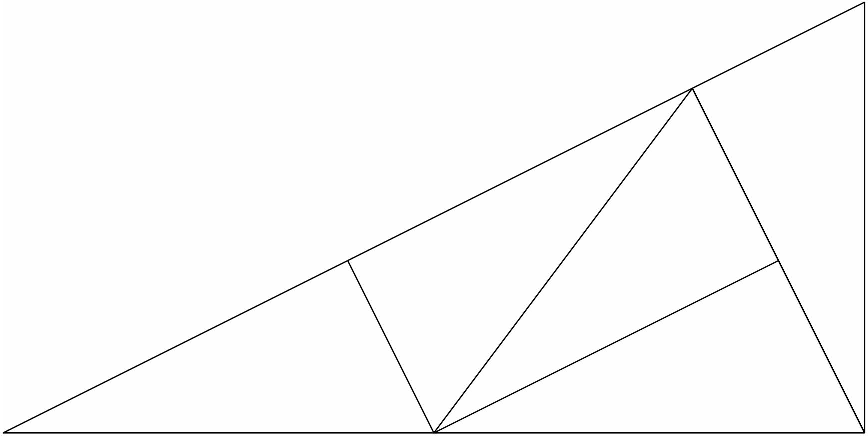
Example: Conway's pinwheel tiling

Start with the triangle

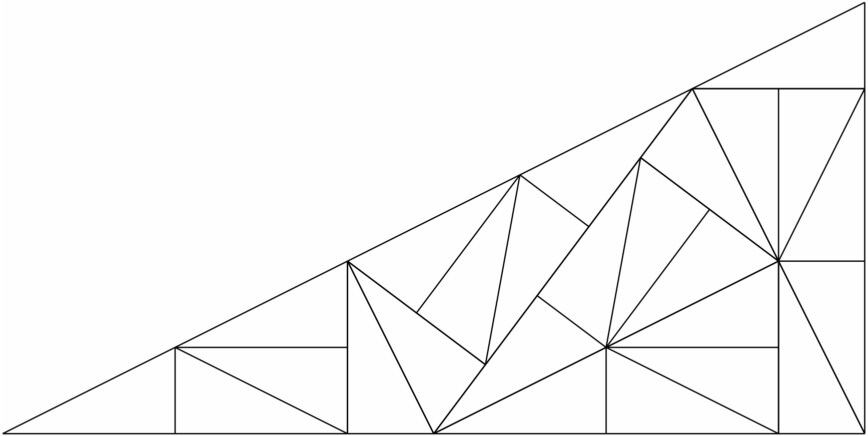


Replace each small triangle with a scaled, rotated copy of the original, and then scale up the result so that each sub-triangle is the same size as the original.

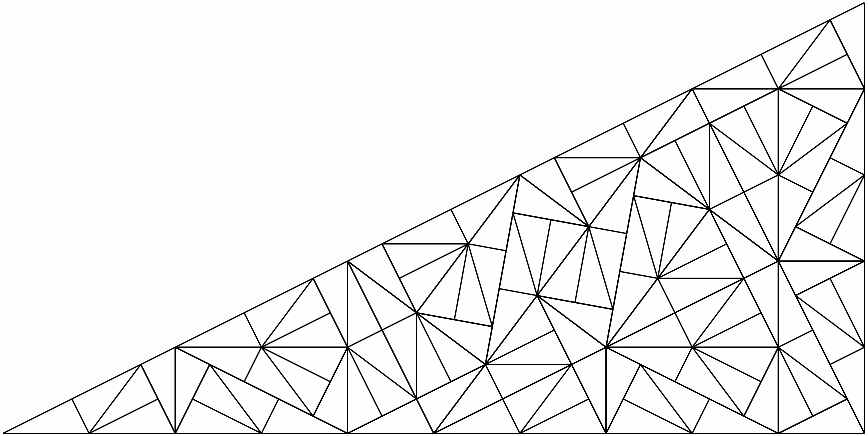
Conway's pinwheel



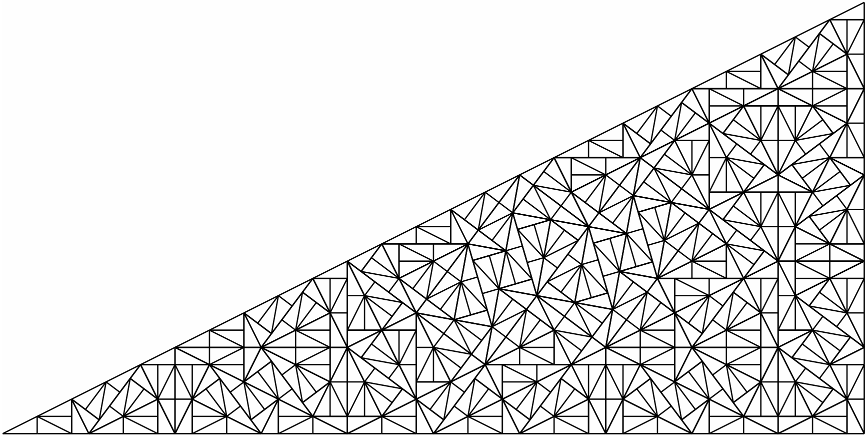
Conway's pinwheel



Conway's pinwheel

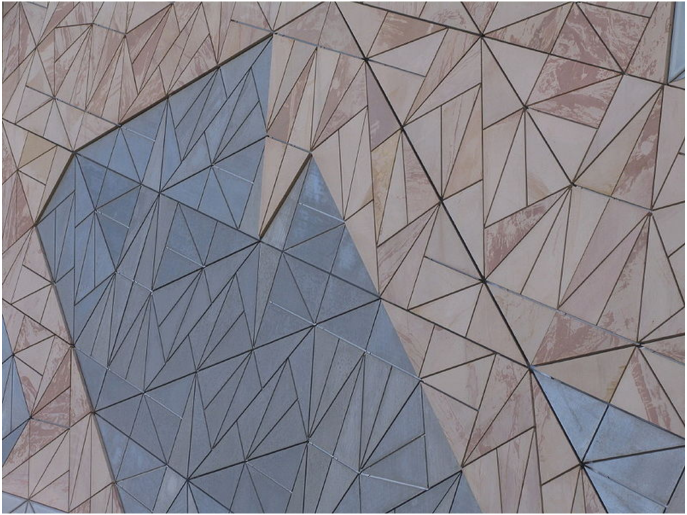


Conway's pinwheel

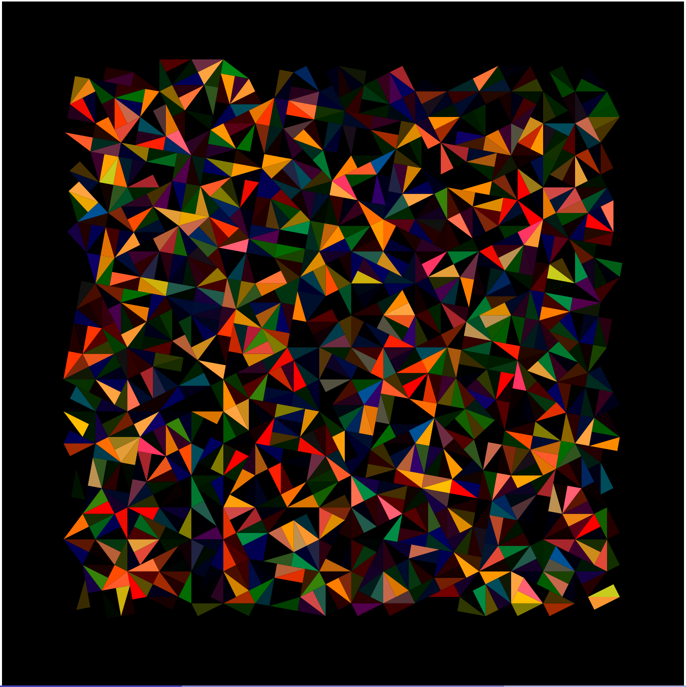


Conway's pinwheel in architecture

Federation square in Melbourne features Conway's tiling



https://en.wikipedia.org/wiki/File:Federation-square-sandstone-facade.jpg



There are many other examples and constructions

- Penrose's tiling
- Nonperiodic monohedral pentagons tilings
- Voderberg spiral tiling
- Random tessellations, *e.g.*, Voronoi diagrams

So where are we now?

- I haven't said much about pattern *formation* yet
- So far we have been developing a *language* to help describe patterns

Takeaways

- Symmetry
- Tessellation
- Irregularity and symmetry breaking

Extra bits

- What about tilings on surfaces other than the plane?
  - sphere
  - higher dimensions
- How would you prove that there are only the regular tessellations I have shown, or that the wallpaper group has only 17 members?

Links

- https://en.wikipedia.org/wiki/Euclidean_tilings_by_convex_regular_polygons
- https://en.wikipedia.org/wiki/List_of_convex_uniform_tilings
- https://www.mathsisfun.com/geometry/tessellation.html
- http://mathworld.wolfram.com/SemiregularTessellation.html
- http://www2.clarku.edu/~djoyce/wallpaper/seventeen.html
- http://xahlee.info/Wallpaper_dir/c5_17WallpaperGroups.html
- https://nrich.maths.org/1341

Further reading I

- Jinny Beyer, *Designing tessellations: The secrets of interlocking patterns*, Contemporary Books, 1999.
- John H. Conway, Heidi Burgiel, and Chaim Goodman-Strauss (eds.), *The symmetries of things*, CRC Press, 2008.
- Frank A. Farris, *Creating symmetry: The artful mathematics of wallpaper patters*, Princeton University Press, 2015.
- Dale Seymour and Jill Britton, *Introduction to tessellations*, Dale Seymour Publications, 1989.