Assignment 1: Due Monday August 19th at 3pm.

Assignments to be handed in through MyUni. Please ensure written assignments are clearly legible. Typed assignments are preferred. Some help may be given in practicals to help get you started with Overleaf/LaTeX in order to present your work well.

Topic: Linear programming basics

1. Translate the following into a Linear Program (showing all detailed coefficients).

   A company wants to meet a production order for porridge. They need to produce at least 200 standard boxes, 100 "low salt" boxes. However, they have some leeway in the exact numbers, provided they can all be fit into a shipping container with capacity 400 boxes. The company's profit on each box is \$1.20 for standard, and \$1.00 for low salt. However, there is only enough salt in the cellars to make an additional 50 standard boxes, or 100 low salt boxes (above the minimum order). Maximise the companies profit.

   You may ignore the requirement to produce integral numbers of boxes.

   NB: you don't need to solve the problem!

2. For the following simple Linear Program

$$
\begin{array}{llrcrcr}
(P) & \max z = & 2x & - & y & & \\
& \text{subject to} & x & + & y & \geq & 6 \\
& & 2x & - & y & \geq & 0 \\
& & 5x & + & 2y & \leq & 27 \\
& & & & x, y & \geq & 0
\end{array}
$$

   (a) Draw the feasible region. Comment on its basic features, *e.g.,* boundedness and convexity.

   (b) Identify the feasible vertices on your drawing. How many are there.

   (c) Calculate the objective at each feasible vertices.

   (d) What is the optimal solution?

   (e) Comment on the value of the objective function at the other two vertices.

3. Imagine that the following problem is a real-world optimisation, where x is the area of orange orchards we plant, y the area of grape vines, and we are subject to constraints and a maximisation labelled below.

$$
\begin{array}{llrcrcrl}
(P) & \max z = & 2x & + & 3y & & & \\
& \text{subject to} & x & + & y & \leq & 120 & \text{; area constraint} \\
& & 2x & + & y & \leq & 100 & \text{; water constraint} \\
& & & & x, y & \geq & 0 &
\end{array}
$$

   Make some informed estimates of what these express, and write out the optimisation problem in words. Try to make your description concise.

   You do not need to solve the problem.

4. In MATLAB, discuss which of the following are good variable names
   (a) `x`
   (b) `12men`
   (c) `happiness%`
   (d) `temp`

5. (a) Write down the definition of the kernel or null space of a matrix A.
   (b) Write down the definition of the rank of A.
   (c) What does it mean to say that $\text{rank}(A) + \text{nullity}(A) = n$

[20 marks]