Assignment 3: Due Friday Sept 6th at 3pm.

Assignments to be handed in through MyUni. Please ensure written assignments are clearly legible. Typed assignments are preferred. Some help may be given in practicals to help get you started with Overleaf/LaTeX in order to present your work well.

Questions about Simplex, duality, complementary slackness, etc.

1. Consider the constraints:

$$\begin{array}{rrcrcr} & x_1 & + & x_2 & \leq & 1 \\ - & x_1 & - & x_2 & \leq & -1 \end{array}$$

   (a) Reduce these to a single constraint.

   (b) Imagine that these constraints are part of a larger problem (with n variables and some other constraints). Use this reduction to help explain the affect of the pair of constraints on the dimension of the feasable set.

2. Consider the LP

$$\begin{array}{lrcrcrcrcr} (P) & \max z & = & x_1 & - & x_2 & + & 3x_3 & & \\ & \text{subject to} & & 2x_1 & + & x_2 & + & 5x_3 & \leq & 6 \\ & & - & 3x_1 & - & 2x_2 & + & 4x_3 & \leq & -3 \\ & & & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0 & & \end{array}$$

   (a) Write down the dual (D) of (P).

   (b) Solve (P) using Simplex.
   You can use Matlab's `linprog` to check your result, but please write out a full solution as you might have to in an exam. Be sure to indicate pivot locations.

   (c) Find the optimal solution of the dual (D).

   (d) Verify the Complementary Slackness Relations (CSRs).

3. Consider the LP

$$
\begin{array}{llrcrcrcl}
(P) & \max z & = & - & x_1 & + & 2x_2 & - & x_3 \\
 & \text{subject to} & & & 2x_1 & + & x_2 & + & 3x_3 \leq 2 \\
 & & & & x_1 & + & 4x_2 & + & 2x_3 \leq 4 \\
 & & & & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0
\end{array}
$$

   (a) Formulate the Dual (D) of (P).

   (b) Establish the Complementary Slackness Relations, for the Primal (P).

   (c) Use the Complementary Slackness Relations (CSRs) for the Primal (P), to find the optimal solution of the dual (D), given that the primal (P) has optimal solution $x_1^* = x_3^* = 0,\ x_2^* = 1$. (No marks will given for Simplex Method solution.)

4. Count the number of operations in the following pseudo-code

```
for i=1 to n
   for j=1 to i
       x(i,j) = i*j + 1;
   end
end
```

5. Reduce the following Big-O notations to a simpler form
   (a) $O(5n)$
   (b) $O(2n + n^2)$
   (c) $O(\log n + n)$
   (d) $O(n)O(\log n)$

[20 marks]