Optimisation and Operations Research

Lecture 14: ILPs in Matlab and AMPL

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Section 1

Integer Programming: Matlab

MATLAB `intlinprog`

Similar to the `linprog` command in Matlab for linear programs that have variables which can take on real solutions, there exists a command `intlinprog` for those linear programs that are also constrained to have variables to be *integer*, *i.e.*, which can only take on the values 0 or 1.

That is, intlinprog solves binary linear programming problems of the form

$$\min_{\mathbf{x}} f^T \mathbf{x}, \quad \text{such that} \quad \begin{cases} A\,\mathbf{x} & \leq & \mathbf{b} \\ A_{eq}\,\mathbf{x} & = & \mathbf{b}_{eq} \\ & & \text{some } x_i \text{ integer} \end{cases}$$

where, f, $\mathbf{b}$, and $\mathbf{b}_{eq}$ are vectors, A and A_{eq} are matrices, and some of the the variables are required to be integers.

MATLAB `intlinprog` example

Commands: for Binary program below

```
> f = [-9; -5; -6; -4];
> A = [6,3,5,2; 0,0,1,1; -1,0,1,0; 0,-1,0,1];
> b = [9; 1; 0; 0];
> Aeq = [];
> beq = [];
> intcon = [1,2,3,4];
> ub = ones(4,1);
> lb = zeros(4,1);
> x = intlinprog(f,intcon,A,b,Aeq,beq,lb,ub)
```

Output:

```
x = 1
    1
    0
    0
```

When to use intlinprog

- Don't cheat
  - use this to check solutions
  - but solve them the way required in your assignments
- Yes, you can use it in your project
  - but display understanding
  - show alternatives
- In general
  - you still need to construct matrices and vectors which is awkward
  - you need to write every constraint, even if they fit a pattern
  - you still need explicit (closed form) constraints and objective functions
  - Matlab doesn't tell you much about how it does it, and what its limitations are
    - ★ we know the problem might be NP-complete, so this could be an issue

Section 2

AMPL

Practical Optimisation

Matlab is all very well, but what do real optimisers do?

Mathematical Modelling Languages

- There are computer languages designed specifically for optimisation
  - they allow natural expression of LPs, etc.
  - they link to multiple *backends* to solve the problem
  - they separate the *model* from the *data*
  - they link to other tools, *e.g.*, databases, spreadsheets, ...
  - support reuse
- Common examples:
  - *AMPL*
  - GAMS
  - AIMMS
  - MPS (not really a modelling language, but is used for standard input)
  - ...

AMPL = A Mathematical Programming Language

- Why AMPL?
  - I like it
  - it's one of the most commonly used (Neos says 59%)
  - lots of backends
  - free student version
- History
  - designed 1985 by Robert Fourer, David Gay and Brian Kernighan
  - 2003 AMPL Optimization LLC
  - 2012 INFORMS Impact Prize

Resources

- **The** AMPL Book, "AMPL: A Modelling Language for Mathematical Programming", R.Fourer, D.M.Gay and B.W.Kernighan
  - Online `http://ampl.com/resources/the-ampl-book/`
- Other tutorials
  - `www.ieor.berkeley.edu/~atamturk/ieor264/samples/ampl/ampldoc.pdf`
- Download your own (student) copy of AMPL from
  - AMPL `http://ampl.com/try-ampl/download-a-free-demo/`
  - backends from the same place or others
    - there are other backends (we are using lpsolve, and I notice they don't have a direct link to this anymore).
- Online solver NEOS `http://www.neos-server.org/neos/` `https://neos-server.org/neos/solvers/lp:Gurobi/AMPL.html`

Example

Example

LP

$$
\begin{array}{rcrcrcr}
\max z & = & 3x_1 & + & 2x_2 & & \\
\text{subject to} & & & & & & \\
 & & 2x_1 & + & x_2 & \leq & 5 \\
 & & -x_1 & + & 4x_2 & \leq & 3 \\
 & & & & x_i & \geq & 0
\end{array}
$$

AMPL file `example.mod`

```
var x{i in 1..2} >= 0;

maximize z: 3*x[1] + 2*x[2];

subject to c1: 2*x[1]+1*x[2]<=5;
subject to c2:  -x[1]+4*x[2]<=3;
```

Example

Start ampl and then type

Example

```
# commands to run in AMPL
ampl: reset;                  # get rid of old data
ampl: model example.mod;      # choose our model
ampl: option solver lpsolve;  # set the backend
ampl: solve;
ampl: display x;              # output the results
```

- You could just type these commands at the AMPL prompt
- Could also run them from a script
- Remember the semi-colons.

Programming Style

Mixes

- Imperative:
  - sequences of commands to execute
  - focus on *how* to perform the task
- Declarative:
  - describe the problem, not how to solve it
  - focus on *what* the task should achieve
- Interpreted:
  - executed in source code form
  - can interact with interpreter (like Matlab)

Models

- Variables

```
var x{i in 1..2} >= 0;
```

  - or could have named variables, *e.g.*, `amount_of_paint`
  - and we can have *sets*, and other constructs

- Objective

```
maximize z: 3*x[1] + 2*x[2];
```

  - we can put a wide range of mathematical expressions here

- Constraints

```
subject to c1: 2*x[1]+1*x[2]<=5;
subject to c2:  -x[1]+4*x[2]<=3;
```

  - we can put a wide range of mathematical expressions here
  - constraints have names, *e.g.*, `c1`, which could use later

Another Example

- Index values can be from an arbitrary set
- Coefficients and variables can be specified as vectors or matrices

Example

```
set possibilities := {"A", "B", "C"};

param a{possibilities};
param b;
param c{possibilities};

var x{possibilities} integer;

maximize profit: sum{i in possibilities} c[i]*x[i];

subject to limit1: sum{i in possibilities} a[i]*x[i] <= b;
subject to limit2{i in possibilities}: 0<= x[i] <= 1;
```

Data and Model Separation

- Why separate data and model?
  - model might actually be very small when you remove repeated bits
    - $x_i \geq 0$ for all i
  - model might be static, but data changes
    - *e.g.*, prices change
  - data might be in another file
    - *e.g.*, spreadsheet or database
  - conceptually easier to understand
- What is separation
  - model shows mathematical structure
  - data fills in the coefficients, which are called *parameters*
- We use notation much like standard mathematical notation

Example: model

Example (Example model file)

```
## Introduction to AMPL - A Tutorial, by Kaminsky and Rajan
##   Example 2
param n;
param t;
param p{i in 1..n};
param r{i in 1..n};
param m{i in 1..n};

var paint{i in 1..n} >= 0 integer;

maximize profit: sum{i in 1..n} p[i]*paint[i];

subject to time: sum{i in 1..n} (1/r[i])*paint[i] <= t;
subject to capacity{i in 1..n}: 0 <= paint[i] <= m[i];
```

Example: data

Example (Example data file)

```
## Introduction to AMPL - A Tutorial, by Kaminsky and Rajan
##    Example 2
param n:= 2;
param t:= 40;

param p:= 1 10
          2 15;
param r:= 1 40
          2 30;
param m:= 1 1000
          2 860;
```

Example

Example (Example commands)

```
# commands to run in AMPL
ampl: reset;                    # get rid of old data
ampl: model test.mod;           # choose our model
ampl: data test.dat;            # specify data
ampl: option solver lpsolve; # set the backend
ampl: solve;
ampl: display paint;            # output the results
```

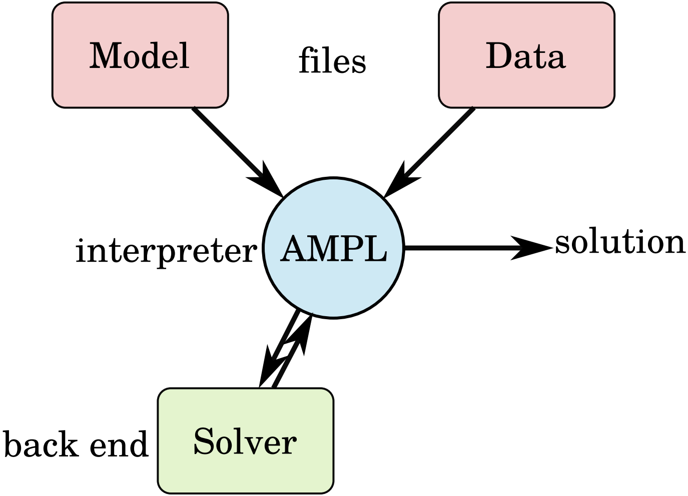
Note we input the model before the data, and we reset first, to clear any old definitions that might conflict.

Data and Model Separation

Note that

- n is set in the data
  - we can change the number of types of paint easily
- The coefficients of the constraints, and the objective are set in the data
  - we can change these as the market changes
  - we could choose more meaningful names for everything
- The expression of all the constraints is done very concisely
  - makes it easier to get it right (less typos)

Data and Model Separation



Advanced AMPL

- We can go way beyond this
  - all sorts of constraints
  - all sorts of objective
  - general sets of objects
  - 2D arrays of parameters
  - data from files
- Solvers (backends)
  - lpsolve
  - CPLEX
  - MINOS
  - Gurobi

  Each can handle different size problems, and different types of constraints (*e.g.*, MINOS can't do Integer problems).

Where to next?

- We will use AMPL in some practical questions
  - you'll get some more help
  - you might need to read some of the other available resources to fill in gaps
  - I may have some more examples in lectures
- You can use it to solve some assignment questions or in your project
  - but read questions carefully – some expect you to use it, and other ask not
  - and make sure you understand the results
- There will be a question in the Exam

Takeaways

- Matlab has a solver for ILPs
  - it's useful when we want an integrated environment to create, solve, and visualise our problem
- AMPL (or another modelling language) is the way most industrial mathematicians *should* approach big problems
  - it's natural
  - multiple backends
  - separation of data and model is very valuable
- We're going to spend some time now to understand how these might work

Further reading I