Tutorial 2

Make sure you prepare these BEFORE the class.

Solutions will be handed out at the tutorial. They will not be put on MyUni.

1. **Translation:** Consider the following *portfolio management* problem. A bank has \$1 million to invest in variety of bonds offered by the government and other agencies. Each bond has a *rated quality*, and an *after-tax yield*, and a *years to maturity* (how long the investment is committed). The portfolio manager must try to maximise the return on investment, but must also meet other criteria:

   1. the average quality of bonds cannot be worse than 1.5 (note that for quality, a low number corresponds to high-quality)
   2. the average years to maturity should not exceed 4 years.

   Assuming there were 4 possible bonds

   (a) What are the variables? *Hint: define variables* x_1, x_2, x_3 *and* x_4.

   (b) What is the objective?

   (c) Write a series of linear constraints. *Hint: there should be three.*

   (d) What are the bounds on the variables?

2. **Interpretation:**

   Imagine we start with a LP

$$
\begin{array}{lrcrcrcl}
\max\ z = & 6x_1 & + & 14x_2 & + & 13x_3 & & \\
\text{subject to} & & & & & & & \\
& \frac{1}{2}x_1 & + & 2x_2 & + & x_3 & \leq & 24 \\
& x_1 & + & 2x_2 & + & 4x_3 & \leq & 60 \\
& & & & & x_i & \geq & 0
\end{array}
$$

   which we put into standard equality form (adding slack variables), and then into the tableau

x_1	x_2	x_3	x_4	x_5	z	b	basic variable
1/2	2	1	1	0	0	24	x_4
1	2	4	0	1	0	60	x_5
−6	−14	−13	0	0	1	0	

   We perform Simplex, and end up with the Tableau

x_1	x_2	x_3	x_4	x_5	z	b	basic variable
1	6	0	4	−1	0	36	x_1
0	−1	1	−1	1/2	0	6	x_3
0	9	0	11	1/2	1	294	

   The optimal solution is therefore $\mathbf{x}^* = (36, 0, 6)$, with $z^* = 294$

   (a) How close to equality are the original constraints at this solution?

   (b) Intepret that "closeness" in the light of the value of the slack variables.

   (c) If we were to increase one of the constraint values, say $60 \to 61$, we could increase one of the slack variables – which one and by how much?

   (d) Use the final row of the tableau to estimate the potential affect of this on the value of z^*

3. **Calculations:** Consider the LP with the Simplex Tableau:

x_1	x_2	x_3	x_4	x_5	x_6	z	b
0	1	−2	0	3	1	0	3
1	2	4	0	1	0	0	4
4	−2	1	1	1	0	0	2
−3	−1	−5	0	0	7	1	10

(a) Explain why each of the following positions would not be a suitable choice for the next pivot position, if the Simplex Method were to be applied to the above tableau.
(i) Row 1, Column 1
(ii) Row 1, Column 3
(iii) Row 3, Column 4
(iv) Row 2, Column 1
(v) Row 2, Column 8
(vi) Row 4, Column 3

(b) Nominate two distinct entries that *could* be selected as suitable pivot positions for the Simplex Method.

(c) What happens to the value of the objective function if you pivot in Column 5?

4. **Proof of the week:**
Show that the following set of constraints is unbounded (no calculation is necessary).

$$\begin{array}{rcrcrcl} 3x_1 & - & 3x_2 & + & 5x_3 & \leq & 50 \\ x_1 & & & + & x_3 & \leq & 10 \\ x_1 & - & x_2 & + & 4x_3 & \leq & 20 \\ & & & & x_i & \geq & 0 \end{array}$$

Without calculation, comment on the maximum of $z = 20x_1 + 10x_2 + x_3$, subject to these constraints.