Research Tools and Methods for the Mathematical Science

Lecture 10: Mathematics as an Art

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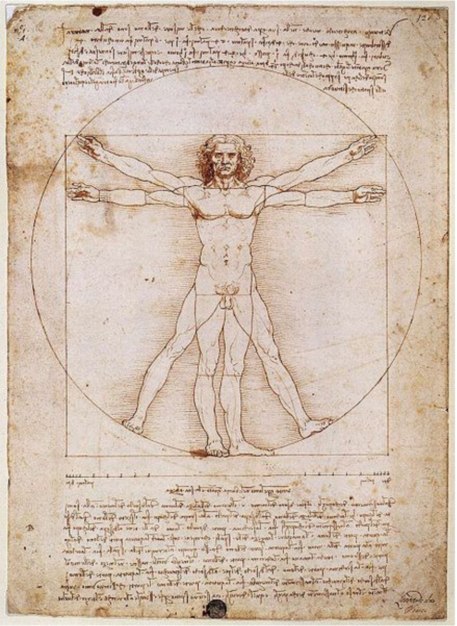
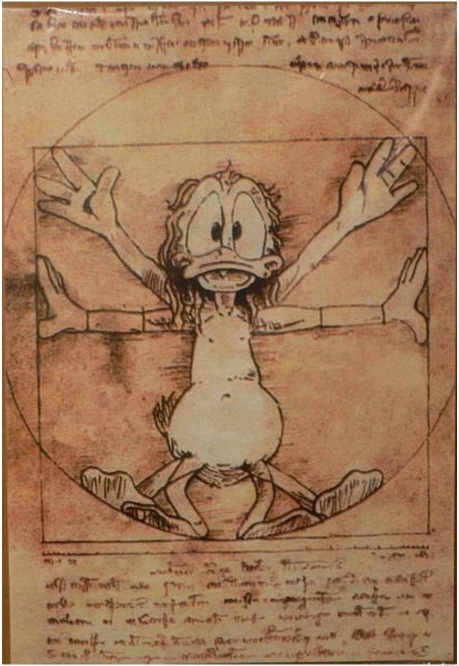
Proof by Obviousness: "The proof is so clear that it need not be mentioned."

> A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.
>
> *G.H. Hardy, A Mathematician's Apology*

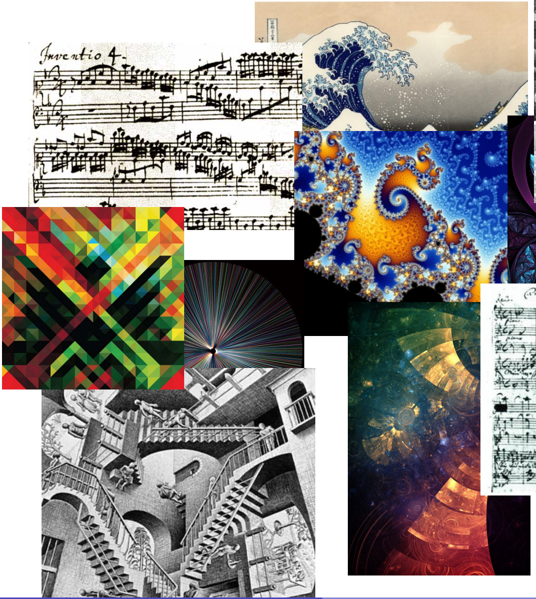
Mathematics is an Art

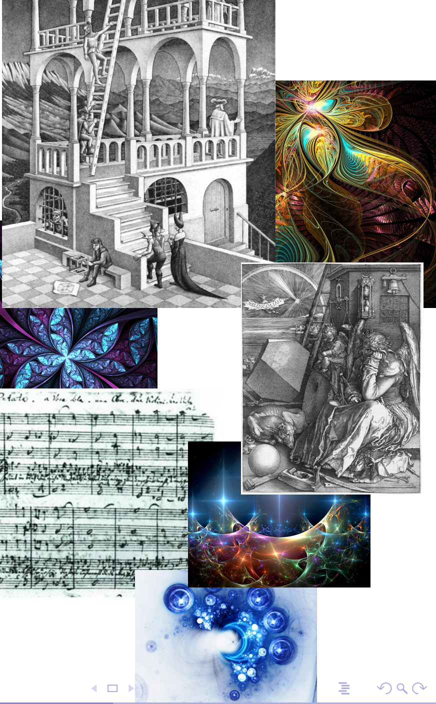




Maths and Art

Usually people mean art influenced by math





But maths is an art in itself

Truth = beauty

$$a^n + b^n = c^n$$

$$e^{i\pi} = -1$$

$$\oint_C (L\,dx + M\,dy) = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right)\,dx\,dy$$

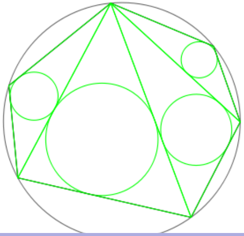
$$\sqrt{2} \neq p/q$$

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}}$$

$$\mathcal{FT}\left(e^{-\pi t^2}\right) = e^{-\pi s^2}$$

$$\phi = \frac{\sqrt{5}+1}{2}$$

$$\frac{d}{dx} \int_a^x f(t)\,dt = f(x)$$



Often the emphasis is on "pure" mathematics [Loc09]

Maths as an Art

Why are numbers beautiful? It's like asking why is Beethoven's Ninth Symphony beautiful. If you don't see why, someone can't tell you. I know numbers are beautiful. If they aren't beautiful, nothing is.

Paul Erdös

Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show.

Bertrand Russell (1872-1970), The Study of Mathematics

The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful.

J.H.Poincare (1854-1912), (cited in H.E.Huntley, The Divine Proportion, Dover, 1970)

Maths as an Art

Mathematics, as much as music or any other art, is one of the means by which we rise to a complete self-consciousness. The significance of Mathematics resides precisely in the fact that it is an art; by informing us of the nature of our own minds it informs us of much that depends on our minds.

J.W.N.Sullivan (1886-1937), Aspects of Science, 1925.

The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics.

G. H. Hardy (1877 - 1947), A Mathematician's Apology, Cambridge University Press, 1994.

Maths as an Art

You have no idea how much poetry there is in the calculation of a table of logarithms!

Gauss, Of Men & Numbers, Jane Muir, Dover, 1996

Characteristic of Weyl was an aesthetic sense which dominated his thinking on all subjects. He once said to me, half-joking, "My work always tried to unite the true with the beautiful; but when I had to choose one or the other, I usually chose the beautiful." (Herman Weyl (1885-1955)).

F.Dyson, in Nature, March 10, 1956

The mathematician is only complete in so far as he feels within himself the beauty of the true.

Goethe via O. Spengler, in J. Newman, The World of Mathematics, Simon & Schuster, 1956

A mathematician who is not at the same time a bit of a poet will never be a full mathematician.

Weierstrass via O. Spengler, in J. Newman, The World of Mathematics, Simon & Schuster, 1956

Applied Maths as an Art

- I want to talk about applied mathematics, because that's what I do
- The real art of applied maths is in modelling
  - there is no right way to model
  - its about balancing tradeoffs
  - often exploratory
    - we need to understand the system to model it, but the point of the model is to gain insight into the system
  - guided by an esthetic sense
    - an effective model is often pleasing (not just because its effective)

Mathematical Modelling

Goal: Answer questions about real-world phenomena
Choices:

- deterministic or stochastic
- discrete or continuous
- linear or non-linear
- closed or open loop
- equilibrium/homogeneous or dynamic/heterogeneous
- micro-scale or macro-scale
- dimensionality of parameter space
- specifics (Gaussian vs heavy-tailed noise)

It's really a choice about what you include and what you exclude from your model, so how do you choose?

Mathematical Modelling

Goal: Answer questions about real-world phenomena
Requirements:

- **simplicity:**
  - tractability (can we do analysis?)
  - measurability (can you measure the parameters? testable?)
  - generalisability (can they predict? can it extend beyond current scope?)
  - insight (does it teach us something useful?)
- **accuracy:**
  - fit to data (statistics)
  - assumptions (explicit/implicit)
  - consistency
  - critical features (physics)

"simplicity" and "accuracy" are often in conflict – what is the right tradeoff between the two?

Mathematical Modelling

Everything Should Be Made as Simple as Possible, But Not Simpler.
Albert Einstein

With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.
Attributed to von Neumann by Enrico Fermi, as quoted by Freeman Dyson in "A meeting with Enrico Fermi" in Nature 427 (22 January 2004) p. 297

Model building is the art of selecting those aspects of a process that are relevant to the question being asked.
J.H Holland

When scientists must choose between competing theories, two men fully committed to the same list of criteria for choice may nevertheless reach different conclusions.
Thomas Kuhn

If its an art, how do I do it well?

- This question is impossibly hard to answer
- But there are ways to think about it that help
  - think about impact
  - learn lots, work hard
  - work efficiently
  - balance long-term and short-term goals
  - open doors
  - think about your process – meta-think
  - be nice to administrators
- Luck is helpful, but not the key [Ham86]

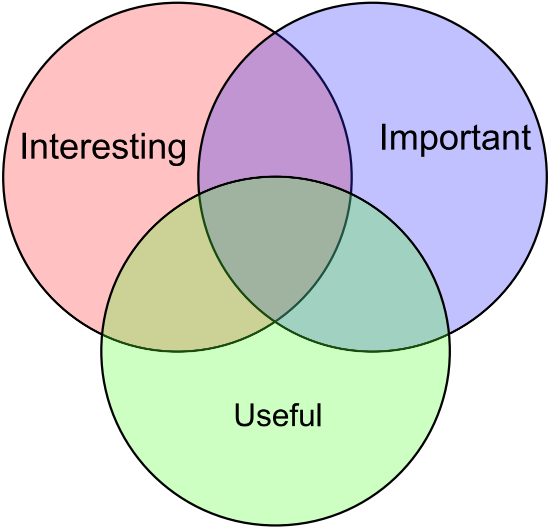
What is "impact"

- Influencing the ideas of others
  - small number of people in a big way
  - lots of people in a small way
- These days, its often measured using metrics
  - citations
  - journal impact factors
  - eigen-factors

> When a measure becomes a target, it ceases to be a good measure.
>
> *Charles Goodhart's Law*

- Don't be cynical about impact
  - work on interesting, important, useful problems
    - ★ interesting to you and others
    - ★ useful (not necessarily applied) == connected
    - ★ important = ?
  - publish in the "right" place
    - ★ good journals are OK
    - ★ the right journal is better
  - sell your work

Obligatory Venn Diagram



Pick two?

Learn lots, Work hard

> Knowledge and productivity are like compound interest.
> *Hamming's Law [Ham86]*

- The more you know, the more you can do, and faster
- You don't know in advance what will be useful
- Has to be efficient, planned work

Balance long-term and short-term goals

- Long-term
  - important problems aren't easy
  - shows some vision of your future
  - too easy to get sucked down a rabbit hole
- Short-term
  - need to get runs on the board
  - keep you interested
  - too much on these, and you never see the forest

Open doors

Work with open doors [Ham86]

- It's not just about letting people in
  - collaboration is good!
- It's also about openness to ideas

Think about your process

- Don't just work
- Reflect on what works, and what doesn't
  - how will you get better?
  - how will you teach?

Be nice to admin

> The lesson I learned in Cairo still applies. The only way to deal with bureaucrats is with stealth and sudden violence.
> *Boutros Boutros-Ghali, 1993*

- Don't treat administrators and assistants as enemies
  - they can help you or hinder you
- Think about what they want
  - get things to them on time
  - how can you make them look good

Some more hints

- Every Mathematician has only a few tricks [Rot97]
  - expand your tricks
- Feynman method [Rot97] – keep a dozen favourite problems constantly in mind and test new tricks you learn against them, say if you are at a seminar.
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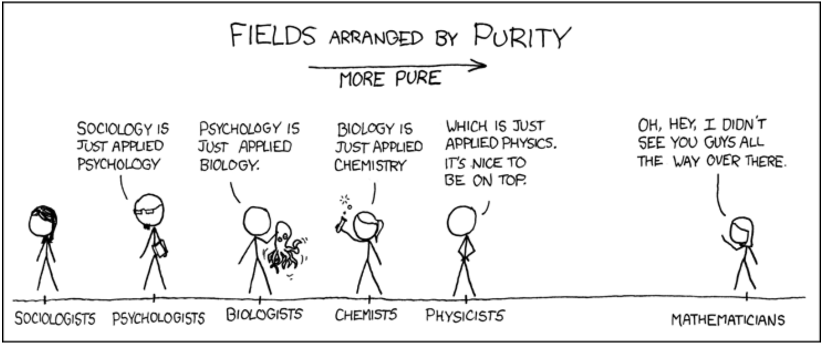

http://xkcd.com/435/

Summary

- Research is not a science
  - no hard and fast rules
  - just hints
- Good luck!

Assignment

Keep working on your 3 minute talk for next week.
http://www.adelaide.edu.au/3mt/

Further reading I

- Ronald T. Azuma, *"So long, and thanks for the Ph.D.!" a.k.a. "Everything I wanted to know about C.S. graduate school at the beginning but didn't learn until later."*, 2003, http://www.cs.unc.edu/~azuma/hitch4.html.
- Richard Hamming, *You and your research*, 1986, http://www.inf.ed.ac.uk/research/programmes/hamming/.
- P. Lockhart, *A mathematician's lament*, Bellevue Literary Press, 2009.
- Gian-Carlo Rota, *Ten lessons I wish I had been taught*, Notices of the AMS **44** (1997), no. 1, 22–25, http://alumni.media.mit.edu/~cahn/life/gian-carlo-rota-10-lessons.html.