

Tutorial 1: Wednesday 1st August

Revision:

1. Basic Skills

- Draw a diagram showing the general shape of a cubic. Now draw one for a quartic, a quintic and a polynomial of degree 6. What sort of degenerate cases are there?
- If \mathbf{x} and \mathbf{y} are two given vectors, and $t \in [0, 1]$, what does $t\mathbf{x} + (1 - t)\mathbf{y}$ (or $\mathbf{y} + t(\mathbf{x} - \mathbf{y})$) represent geometrically?
- Write the definition of a partial derivative, and find the partial derivatives with respect to x of
 - (a) $f(x, y) = x^2 + y^4 - 3x^4y$
 - (b) $f(x, y, z) = x \sin(2x + yz)$
 - (c) $f(x, y, z) = (x^2 - y)^2 e^z$
- Find the gradient $\nabla f(\mathbf{x})$ for the following functions f .
 - (a) $f(x, y) = x^2 + y^4 - 3x^4y$
 - (b) $f(x, y, z) = x \sin(2x + yz)$
 - (c) $f(x_1, x_2, x_3) = (x_1^2 - x_2)^2 e^{x_3}$
 - (d) $f(\mathbf{x}) = \|\mathbf{x}\|$, where $\mathbf{x} \in \mathbb{R}^n$
- Find the Taylor series expansion for
 - (a) $\cos x$
 - (b) $\ln(1 + x)$
 about the point $x = 0$.

2. Use the multivariable chain rule to find dz/dt where

$$z = 2x^2 + 3xy - 4y^2$$

and

$$x = \cos t, \quad \text{and } y = \sin t.$$

3. Use Taylor's Theorem to derive a polynomial approximation (of at least degree 2) for $f(x, y) = \sin(x + y^2)$.
4. Find the cylinder of largest volume that can be placed inside a sphere of radius 1.