ADELAIDE UNIVERSITY AUSTRALIA

Examination in the School of Mathematical Sciences

Semester 1, 2007

006426	**COMMUNICATIONS NETWORK DESIGN**
	APP MATH 7026

Official Reading Time: 10 mins
Writing Time: 180 mins
Total Duration: 190 mins

ANSWER ALL QUESTIONS
NUMBER OF QUESTIONS: 4 TOTAL MARKS: 100

Instructions

- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.

Materials

- 1 Blue book is provided.
- Calculator without an alphanumeric memory or remote communications capability is permitted.

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO.

1. (a) Perform Dijkstra's Algorithm on the network shown in Figure 1 to find the shortest paths from node 1 to all the other nodes.

Show all your working.

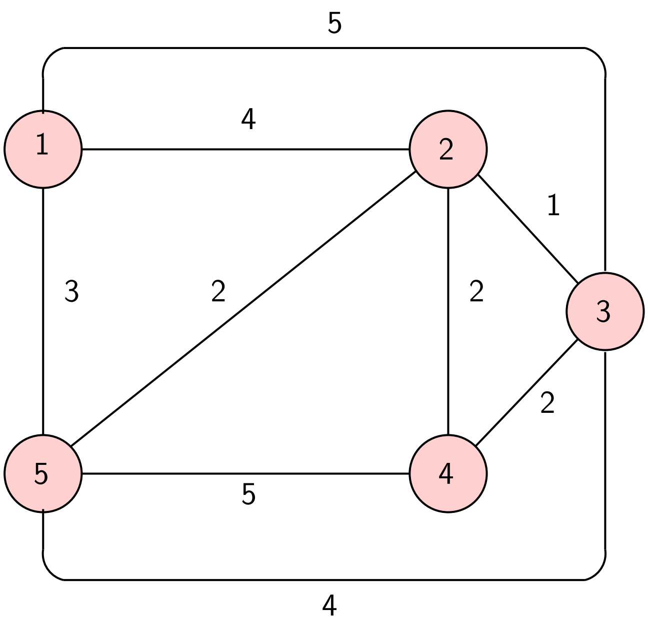

Figure 1: Network and link costs

(b) The Floyd Warshall Algorithm could also be used to find a solution to the above shortest path problem. Explain under what conditions it would end up with the same solution as that given by Dijkstra's Algorithm.

(c) Write down the formal statement of the routing problem for linear separable costs, carefully defining all the terms in your formulation.

(d) What sort of routing results from linear separable costs and why?

(e) We often assume linear separable costs, but give an example of when it might not be a reasonable assumption to make.

[22 marks]

2. (a) Using a sentence or two for each part, describe briefly

   (i) The key difference between circuit switching and packet switching.

   (ii) The robustness principle.

   (iii) The difference between routing and forwarding.

   (iv) The major costs associated with Networking (include both capital and operational costs).

   (v) Why a distributed network is commonly considered superior to a centralised network.

   (b) Using a paragraph or two (no more than a page) describe one of the following

   (i) The end-to-end principle.

   (ii) The post office analogy and the OSI model.

[23 marks]

3. Consider a cost function of the form

$$C(\mathbf{f}) \;=\; \sum_{e\in E} (\alpha_e f_e + \beta_e)$$

and the costs and offered traffic as in Figure 2.

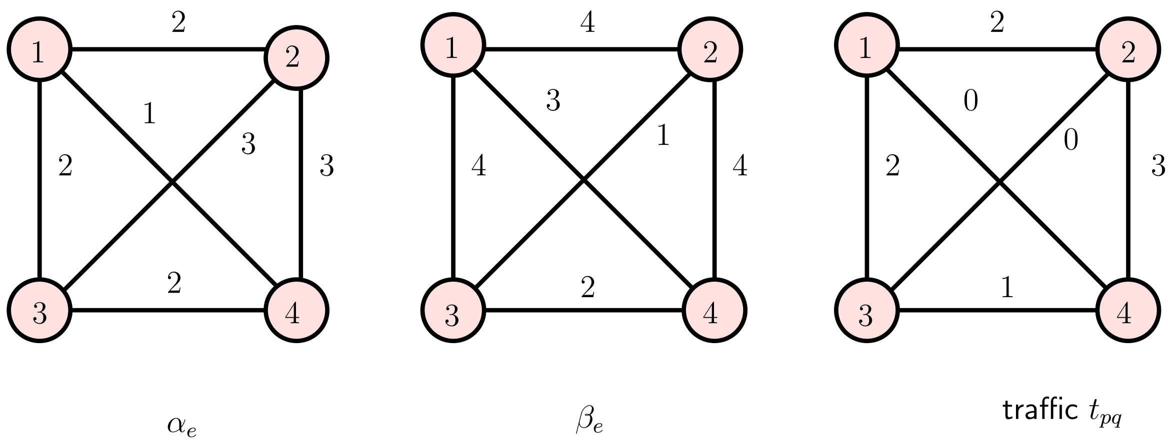

Figure 2: Network costs and traffic

(a) Apply Minoux's Algorithm to find a minimum cost network. **Show all your working.**

(b) In the Budget Constraint Model, we considered the network $G(N,E)$ and cost function

$$v(L) \;=\; \sum_{e\in L} \alpha_e f_e \quad \text{subject to the constraint} \quad \sum_{e\in L} \beta_e \le B\,.$$

(i) Explain carefully how the associated Knapsack Problem

$$\max\left\{ \sum_{e\in E} d_e z_e \Big| \sum_{e\in E} \beta_e z_e \le B, z_e \;=\; 0 \text{ or } 1 \text{ for all } e\in E \right\},$$

$$\text{where} \quad d_e = \Big(\hat{l}_k\big(E\setminus\{e\}\big) - \alpha_e\Big)\, t_k\,,$$

enables us to minimise $v(L)$ in the Budget Constraint Model. Here, $\hat{l}_k\big(E\setminus\{e\}\big)$ is the length of the shortest path for traffic on path k if link e is removed from the set of links E.

(ii) Now perform one step of Branch-and-Bound (that is, to the first branch point) for finding the network (N,L) that minimises the cost given $B = 9$. Calculate the Dionne-Florian lower bound at the end of this step for bounding the partial solutions. **Show all your working.**

(iii) If the requirement changes to $\sum_{e\in L} \beta_e \le 6$, write down directly the network $G(N,L)$ that minimises the cost.

[30 marks]

4. (a) Consider the network in Figure 3 with β_e as specified against each link in the figure and $\alpha_e = 0$ for all e.

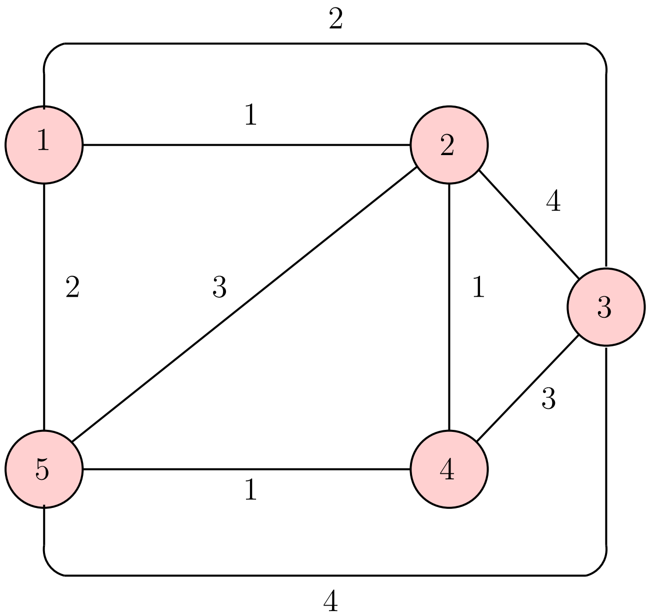

Figure 3: A network of 5 nodes

(i) Find a minimum weight spanning tree using Prim's Algorithm.

Show all your working.

(ii) What are the differences between Kruskal's Algorithm and Prim's Algorithm, and are they both optimal?

(iii) Adding a link to the solution provided by Prim's algorithm in this problem gives us a solution to the travelling salesman problem, that is, a tour. Explain whether or not we can always use Prim's algorithm or maybe Kruskal's algorithm to solve the travelling salesman problem.

(b) Provide a definition of

(i) a cutset.

(ii) a fundamental cutset.

(c) What type of cutsets are used in the Gomory-Hu method and what does this imply with regard to complexity?

(d) Consider now the network shown in Figure 3, where we now assume $\alpha_e = 1, \beta_e = 0$ for all e and the numerical values now represent the offered traffic between the nodes. Choosing node 1 as the initial hub, perform the first iteration of Gusfield's algorithm, showing the network before and after the iteration. **Show all your working.**

[25 marks]