Information Theory and Networks Lecture 10: Sampling with Fair Coins

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> > September 18, 2013



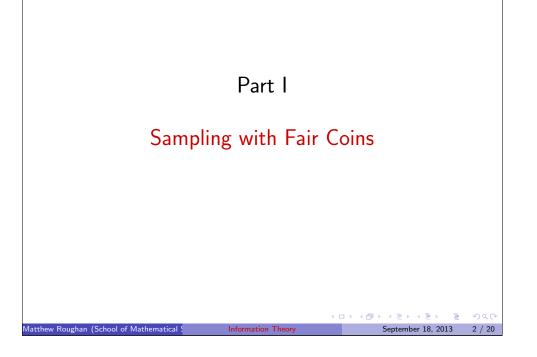
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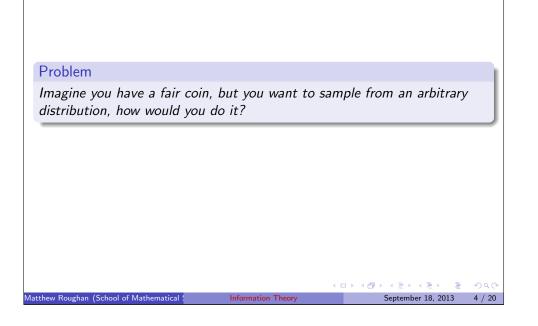
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USA Today has come out with a new survey: Apparently three out of four people make up 75 percent of the population. David Letterman

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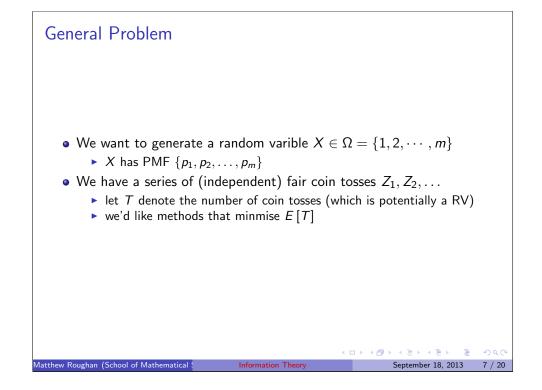
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From [CT91, p.110-116]	

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(e, men probability	_/ ·,
$\wedge = $	a, with probability 2b, with probabilityc, with probability	1/4.
v	a, with probability	1/2,
X with PMF		1 /0
Example: use a sequence of f	fair coin tosses to ge	enerate a random variable
Example 1		

Information Theory 81- 60- Example 1 100 100 100 100 100 100 100 1	Example 1 Example use a sequence of fair cain bases to generate a rendem variable X and PMF $X = \begin{cases} x, with probability 1/2, \\ x, with probability 1/4, \\ c, with probability 1/4, \end{cases}$

Example 1			
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	ation Theory	General Problem
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Example 1			
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Inform 81-60-18 0013-002	Example 1	Example 1

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Problem

How could we go about designing such a tree for a dyadic distribution (one whose probabilities are powers of two)?

Is it related to entropy?

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Example 2			
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Problem	
What about non-dyadic probabilities?	
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	Obviously, we could do powers of three with tempory codes	

Obviously, we could do powers of three with ternary codes, and so on, do lets assume that the probabilities don't all fit some simple power-law.

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Example 3		
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Theorem

The expected number of fair bits E[T] required by the optimal algorithm to generate a random variable X satisfies

$H(X) \leq E[T] < H(X) + 2$

Proof: see [CT91, pp.115-116]

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Example 3			
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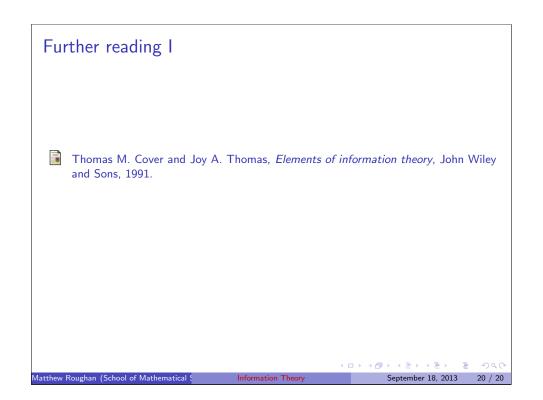
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We could have also looked into Stochastic Computing here – i.e., techniques for doing computation using operations stochastic processes.

Source Coding and 20 Questions Yet another way to think about coding 20 questions: Want to guess a 'fact' — say an experiment's outcome Only allowed Yes/No questions Want to find the most efficient set of questions Obviously, Huffman code is optimal way of generating questions if we know the PMF

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