

**Assignment 7:** Due Monday 20th September at 5pm

Late assignments will not be accepted except by prior arrangement (for a good reason)

**Please include your student number in your handed up work, as Canvas doesn't give this to me automatically.**

1. Given a semiring  $(S, \oplus, \otimes, \bar{0}, \bar{1})$ , define  $M_n(S)$  to be the set of square  $n \times n$  matrices, with elements from  $S$ , such that

- $A \hat{\oplus} B$  is element-wise addition

$$[A \hat{\oplus} B]_{ij} = a_{ij} \oplus b_{ij}$$

- $A \hat{\otimes} B$  is the generalisation of standard matrix multiplication

$$[A \hat{\otimes} B]_{ij} = \bigoplus_{k=1}^n a_{ik} \otimes b_{kj}$$

- and identities of the form, *e.g.*,

$$\mathbf{0} = \begin{bmatrix} \bar{0} & \bar{0} \\ \bar{0} & \bar{0} \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} \bar{1} & \bar{0} \\ \bar{0} & \bar{1} \end{bmatrix}$$

Show that

- $\hat{\otimes}$  is associative; and
- idempotence of  $\otimes$  does not imply idempotence of  $\hat{\otimes}$ .

[5 marks]

2. Write (computer) functions implementing  $\oplus$  and  $\otimes$ , *i.e.*, `oplus(A,B)` and `otimes(A,B)` for  $A, B \in M_n(S)$ , where  $S$  is the Viterbi, or Max-times Semiring.

- Use your functions to find  $A^*$  (by fixed-point iteration or otherwise) the solution  $Y$  to the equation

$$Y = (A \otimes Y) \oplus \mathbf{I}$$

for

$$A = \begin{pmatrix} 0.0 & 0.3 & 0.9 & 0.1 \\ 0.5 & 0.0 & 0.5 & 0.5 \\ 0.9 & 0.2 & 0.0 & 0.1 \\ 0.1 & 0.6 & 0.0 & 0.0 \end{pmatrix}$$

[3 marks]

- Explain the meaning of this result.

[1 mark]

- What does it mean that  $A^*$  is not symmetric?

[1 mark]