Assignment 1: Solutions

TOTAL MARKS: 20

1. • Variables

$$\begin{aligned} x_1 &= \text{number of standard boxes} \\ x_2 &= \text{number of low-salt boxes} \end{aligned}$$

[1 mark]

• Objective: maximise $z = 1.2x_1 + 1.0x_2$ [1 mark]

• Constraints: such that

$$\begin{array}{rcrcll} x_1 & & & \geq & 200, & \text{order constraint} \\ & & x_2 & \geq & 100, & \text{order constraint} \\ x_1 & + & x_2 & \leq & 400, & \text{container-size constraint} \\ 2(x_1 - 200) & + & (x_2 - 100) & \leq & 100, & \text{salt constraint} \\ & & x_1, x_2 & \geq & 0, & \text{non-negativity} \end{array}$$

The last constraint can be rewritten in the form

$$2x_1 \;+\; x_2 \;\leq\; 600, \quad \text{salt constraint}$$

[3 marks]

Alternative approach:

• Variables

$$\begin{aligned} x_1' &= \text{number of standard boxes above the minimum 200} \\ x_2' &= \text{number of low-salt boxes above the minimum 100} \end{aligned}$$

• Objective: maximise $z = 1.2(x_1' + 200) + 1.0(x_2' + 100) = 1.2x_1' + 1.0x_2' + 340$

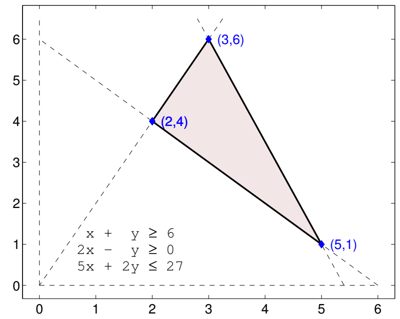
• Constraints: such that

$$\begin{array}{rcrcll} (x_1' + 200) & + & (x_2' + 100) & \leq & 400, & \text{container-size constraint} \\ 2x_1' & + & x_2' & \leq & 100, & \text{salt constraint} \\ & & x_1', x_2' & \geq & 0, & \text{non-negativity} \end{array}$$

The first constraint can be written as

$$x_1' \;+\; x_2' \;\leq\; 100, \quad \text{container-size constraint}$$

2. (a) Sketch of region. It is bounded and convex. [2 marks]



(b) The feasible vertices are shown. [1 mark]

(c) The values of z at the feasible vertices are

vertex	$z = 2x - y$
(3,6)	0
(5,1)	9
(2,4)	0

So z is maximised at $(5, 1)$. [1 mark]

(d) The values at the other two vertices are the same (0) because the boundary line is a contour line for the objective function.

[1 mark]

3. There are some ambiguous components to this problem, e.g., what exactly are the units. You will not be penalised for choosing the wrong units, but you will be for not including units at all.

   An example solution might be:

   > A grower wants to maximise the profits from planting 120 hectares of fields either with oranges or grapes, the profits from which are \$2000 and \$3000 per hectare, respectively. He is also limited by having only 100 kilolitres of water available for irrigation, with each hectare of oranges and grapes taking 2 or 1 megalitre, respectively.

   One mark for objective, one for variables, and one for constraints (including units) [3 marks]

4. In MATLAB , discuss which of the following are good variable names

   (a) `x`

   This is an ambiguous (and hence) poor name in many cases, but if it related to a particular text, where x is used, then it is fine. [1 mark]

   (b) `12men`

   This is an invalid variable name (they can't start with a number) [1 mark]

   (c) `happiness%`

   This is an invalid variable name (the % is interpreted as starting a comment) [1 mark]

   (d) `temp`

   Calling a variable temp should only be done if the variable really does have a very temporary lifetime, *e.g.,* in swapping variable values as in

   ```
   temp = a;
   a = b;
   b = temp;
   ```

   [1 mark]

5. (a) the kernel or null space of a matrix A is the set $Null(A) = \{\mathbf{x} \in \mathbb{R}^n | A\mathbf{x} = \mathbf{0}\}$ [1 mark]

   (b) The rank of A is equivalently

   - the dimension of the row space of A (the vector space spanned by the rows),
   - the maximum number of linearly independent rows of A,

   or the same applied to columns. [1 mark]

   (c) The rank of A is the dimension of its row space and nullity is the dimension of the null space. The result says that the two add to the dimension of the embedding space (the number of columns of the matrix). [1 mark]