Variational Methods and Optimal Control
Class Exercise 1: due before lecture, on Thursday 9th August, 2012

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1. Use the technique of Lagrange multipliers to maximize $V = xyz$ for $x, y, z \geq 0$ subject to the pair of constraints

$$\begin{aligned} xy + yz + zx &= 1 \\ x + y + z &= 3 \end{aligned}$$

2. Maximize $V = x^2 + 2y^2 - z^2$ subject to

$$x^2 + y^2 + z^2 \leq 1$$

3. Which of the following are functionals of the function $y(x)$ (label yes or no).

(a) $y(0) + 4$

(b) $\left.\frac{dy}{dx}\right|_0$

(c) $\min\{y(x) | 0 \leq x \leq 1\}$

(d) $\int_0^1 y \, dx$

(e) $\int_0^\pi \left[\frac{d^n y}{dx^n}\right]^3 f(x) \, dx$

4. Given the L^2-norm $||f||_2 = \sqrt{\int_0^1 f(x)^2 \, dx}$ on the vector space $L^2[0, 1]$, describe (in one sentence) the ε-neigbourhood of the function $y = x$.

5. Find an upper bound for the minimum of the functional

$$J\{y\} = \int_0^1 y^2 y'^2 \, dx,$$

subject to $y(0) = 0$ and $y(1) = 1$ using the trial functions

$$y_\varepsilon(x) = x^\varepsilon,$$

with $\varepsilon > 1/4$. Justify your argument.