# Privacy Preserving Data-Mining 

## and Its Application to Large-Scale Network Measurements

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## Data is key

- Data is the key understanding traffic
- Data is the key to good models
- Data is the key to prediction/planning, anomaly detection, traffic engineering, ...
- Good data is hard to get
- for most people (if not Google)


## Why is data hard to get?

- Companies don't share
- companies don't want to reveal data
- afraid of misuse of data
- afraid it will reveal business secrets
- afraid it will reveal incompetence
- sometimes they are not allowed to
- e.g. privacy legislation [?]
- no particular company sees all the Internet
- the Internet is (by its nature) distributed
- each company can add a perspective


## What's the problem

- How much traffic is there on the Internet?
- the argument is made [?] that lack of such data contributed to the tech-wreck
- regulators need such information
e.g. anti-trust cases
- Detecting distributed attacks
- DDoS (Distributed Denial of Service), Worms/viruses,
- e.g. Worms are easy to detect once they are well under way, but if you want to detect it early, the more data points you have the better.
- but if companies won't share data, how can we collect Internet wide measurements?


## Similar problems elsewhere

- The Center for Disease Control and Prevention (CDC) who have to detect new health threats
- need data from
- hospitals
- insurance companies, airlines, ...
- NGOs (e.g. charities)
- other government bodies
$\square$ data is
- proprietary (e.g. insurance risks)
- protected by privacy legislation
- data-mining community has developed solutions
- secure-distributed computing [?, ?, ?]
- privacy-preserving data-mining [?, ?]


## Trusted third party

- simple answer: a trusted third party

■ independent party (e.g. with no vested interest)

- trusted by all other parties
- collects data, and shares the results
- problems:
- hard to find such parties
- in Internet research anyway
- an exception is the ABS [?]
- Australian Bureau of Statistics collects information about ISPs, including some traffic measurements
- often requires special legislation
- lacks flexibility


## A Couple of problems

Well known problems in the area

- Dining cryptographers
- Millionaire problem

Internet measurement problems

- traffic
- performance


## Dining cryptographers

- $N$ cryptographers are having dinner
- When it is time to pay the bill, the waiter tells them that someone has already paid
- the cryptographers are suspicious by nature (particularly Alice and Bob).
- they suspect the NSA has paid
- not wanting to be compromised by such an association, they need to find out if someone at the table paid, or an external party such as the NSA
- how can they do so, without anyone revealing whether they paid or not?
- of course, the waiter is sworn to secrecy


## Millionaire problem

- Bill Gates and Warren Buffet are trying to decide who should put more money into the Gates foundation (*)
- they want to know who is richer
- But they are feeling rather secretive, and don't want to reveal their true wealth.
- how can they decide?

There are some generic techniques that can help us out

- Secure Distributed Summation (SDS)
- Secure distributed dot product
- Oblivious transfer


## Secure Distributed Summation wiow

Problem: $N$ parties each have one value $v_{i}$ and they want to compute the sum

$$
V=\sum_{i=1}^{N} v_{i}
$$

but they don't want any other party to learn their value.

## SDS algorithm [?]

Assume the value $V \in[0, n]$ (for large $n$ )

```
party 1: randomly generate R~U(0,n)
party 1: compute s}=\mp@subsup{s}{1}{}=\mp@subsup{v}{1}{}+R\operatorname{mod}
party 1: pass s1 to party 2
for i=2 to N
party i: compute si}=\mp@subsup{s}{i-1}{}+\mp@subsup{v}{i}{}\operatorname{mod}
party i: pass si to party i+1
endfor
party 1: compute v}\mp@subsup{v}{N}{}=\mp@subsup{s}{N}{}-R\operatorname{mod}
```

Finally, party 1 has to share the result with the others.
$s_{i}$ will be uniformly randomly distributed over $[0, n]$ and so we learns nothing about any other parties values.

## SDS algorithm



```
party 1: randomly generate }R~U(0,n
party 1: compute }\mp@subsup{s}{1}{}=\mp@subsup{v}{1}{}+R\operatorname{mod}
party 1: pass }\mp@subsup{s}{1}{}\mathrm{ to party }
for i=2 to N
    party i: compute si}=\mp@subsup{s}{i-1}{}+\mp@subsup{v}{i}{}\operatorname{mod}
    party i: pass si to party i+1
endfor
party 1: compute v
```


## SDS algorithm



## SDS algorithm

party 1: randomly generate $R \sim U(0, n)$
party 1: compute $s_{1}=v_{1}+R \bmod n$
party 1: pass $s_{1}$ to party 2
for $i=2$ to $N$
party i: compute $s_{i}=s_{i-1}+v_{i} \bmod n$
party i: pass $s_{i}$ to party $i+1$

endfor
party 1: compute $v_{N}=s_{N}-R \bmod n$

## Applications

- dining cryptographers
- $v_{i}$ equals 1 if a diner paid, zero otherwise, $n=1$, and $V \in\{0,1\}$
- calculating the total traffic on the Internet
- $v_{i}$ is total per ISP
- need some care to avoid double-counting
- Internet health (e.g. by accumulating certain statistics, e.g. packet drops)
- e.g. $v_{i}$ is packet loss percent at each ISP
- use sum to compute (weighted) average
- time series algorithms (either pre- or post-)
- Sketches


## Application to Sketches

Could apply this approach to many sources of data

- number of routers, number of links, or number of links of each type (e.g. OC48, Gig-Ethernet)
- kilometres of fiber, bandwidth-miles of network capacity,
- traffic-miles for carried traffic,
- detailed traffic data (e.g. netflow)
- performance data (packet loss, delay, reordering, ...)

Lots of sorts of data, and in particular for complex data (traffic) the dimensionality of dataset could be very high.

## Application to Sketches

Sketches [?] are an approach to reduce dimensionality of streaming datasets, e.g. Count-Min sketch [?]

- Data: a stream of updates $(a, u)$, where $a \in\{1, \ldots, n\}$ is a key, and $u \in \mathbb{R}$ a value.
- Signal: a vector $v \in \mathbb{R}^{n}$, where for each update ( $a, u$ ), we perform $v_{a}+=u$.
- Sketch: consists of a $d \times w$ array of counts: $c[1,1] \ldots c[d, w]$, and $d$ random hash functions $h_{1}, \cdots, h_{d}:\{1 \cdots n\} \rightarrow\{1 \cdots w\}$, for $w \ll n$
- Update: When an update ( $a, u$ ) arrives, update $c\left[i, h_{i}(a)\right]+=u$ for all $1 \leq i \leq d$.
- Query: When a point query $Q(a)$ arrives, an approximation of $v_{a}$ is given by $\hat{v}_{a}=\min _{i} c\left[i, h_{i}(a)\right]$.


## Application to Sketches

Its almost trivial to extend SDS to sketches:

- agree on common hash functions (and array sizes)
- compute a sketch locally at each party
- use SDS to sum each element in the array
- the point is that given $K$ updates $\left\{\left(a_{i}^{(n)}, u_{i}^{(n)}\right)\right\}_{i=1}^{K}$ from party $n$
$\operatorname{Sketch}\left(\cup_{n=1}^{N}\left\{\left(a_{i}^{(n)}, u_{i}^{(n)}\right)\right\}_{i=1}^{K}\right)=\sum_{n=1}^{N} \operatorname{Sketch}\left(\left\{\left(a_{i}^{(n)}, u_{i}^{(n)}\right)\right\}_{i=1}^{K}\right)$
- we can use the final sketch as needed, e.g. in anomaly detection


## Honest but curious model

- any party could corrupt the total $V$ by inputing incorrect data $v_{i}$
- calculation has implicit assumption of honesty
- let us extend this
- "Honest but curious" security model
- honest: honestly follow protocol
- curious: may perform more operations to try and learn more information (than they were supposed to learn)
- doesn't prevent colluding coalitions
- conditions can be weakened (e.g. honest majority)


## Collusion

- Assume party $j$ and $j+2$ collude
- They know at least $s_{j}$ and $s_{j+1}$
$\square s_{j+1}-s_{j} \bmod n=v_{j}$
- so they can learn the value of $j$
- Various methods of prevention, e.g.
- divide $v_{i}$ randomly into shares $v_{i m}$ such that

$$
\sum_{m} v_{i m}=v_{i}
$$

- sum over $i$ in a different order for each $m$.

$$
\sum_{i=1}^{N} v_{i m}=V_{m}
$$

■ sum $V_{m}$ normally $V=\sum_{m} V_{m}$

## Another applications

ISPs measure one-way inter-provider performance

- inter-provider: many problems occur at the edges
- one-way: inter-ISP routing is asymmetric



## Internet perf. measurement

Experiment and notation:

- send $K_{i j}$ probe packets from ISP $i \rightarrow j$
- sender $i$ notes transmit times $t_{i j}^{(k)}$
- receiver $j$ notes receive times $r_{i j}^{(k)}$
- delay $d_{i j}^{(k)}=r_{i j}^{(k)}-t_{i j}^{(k)}$
- averages:

$$
\begin{gathered}
\bar{D}_{i j}=\frac{1}{K_{i j}} \sum_{k=1}^{K_{i j}} r_{i j}^{(k)}-t_{i j}^{(k)} \\
\bar{R}_{i j}=\frac{1}{K_{i j}} \sum_{k=1}^{K_{i j}} r_{i j}^{(k)}, \quad \bar{T}_{i j}=\frac{1}{K_{i j}} \sum_{k=1}^{K_{i j}} t_{i j}^{(k)}
\end{gathered}
$$

## Internet perf. measurement

- but what if the ISP's don't want other ISPs to be able to make comparisons?
- obviously this limits the type of measures we can make: consider averages across providers, e.g.

$$
\bar{D}_{i}^{\text {out }}=\frac{1}{N-1} \sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{D}_{i j}
$$

- limits what data can be shared:
- ISP's can't share individual measurements $r_{i j}^{(k)}$ or $t_{i j}^{(k)}$


## SDS to the rescue

$$
\begin{aligned}
\bar{D}_{i}^{\text {out }} & =\frac{1}{N-1} \sum_{\substack{j=1 \\
j \neq i}}^{N} \frac{1}{K_{i j}} \sum_{k=1}^{K_{i j}}\left[r_{i j}^{(k)}-t_{i j}^{(k)}\right] \\
& =\frac{1}{N-1}\left[\sum_{\substack{j=1 \\
j \neq i}}^{N} \bar{R}_{i j}-\sum_{\substack{j=1 \\
j \neq i}}^{N} \bar{T}_{i j}\right]
\end{aligned}
$$

- $\sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{T}_{i j}$ is already known by $i$
- $\sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{R}_{i j}$ calculate using SDS and give $i$ the result


## But wait...

What happens when packet are lost?

- we can't compute $\bar{D}_{i}^{\text {out }}$ without censoring the transmit times for the lost packets
- we can't tell other ISPs when packet are lost
- this would reveal a great deal about performance
- we can't include straight sequence numbers in packets
- these would allow statistical inference


## Secure Dot Product (SDP) [?]

- Alice has a vector a, and Bob has a vector $\mathbf{b}$.
- They want to compute

$$
\mathbf{a .} \mathbf{b}=\sum a_{i} b_{i}
$$

without revealing any $a_{i}$ or $b_{i}$ to each other

- can't just return a $\cdot \mathbf{b}$ because some choices of a would reveal parts of $\mathbf{b}$.
so split the solution

$$
V_{a}+V_{b}=\mathbf{a} \cdot \mathbf{b}
$$

and return $V_{a}$ to Alice and $V_{b}$ to Bob.

## Solution

- add a randomly chosen packet ID to each packet:
- ID chosen randomly from $\{1,2, \ldots, L\}$ where $L \geq K_{i j}, \forall i, j$
- create Identity vectors (at receivers)
$I_{i j}^{(k)}= \begin{cases}1, & \text { if the packet with ID } k \text { from } i \text { to } j \text { is received, } \\ 0, & \text { otherwise. }\end{cases}$
- now the calculation is

$$
\bar{D}_{i}^{\text {out }}=\frac{1}{M_{i}} \sum_{\substack{j=1 \\ j \neq i}}^{N}\left[\sum_{k=1}^{L} I_{i j}^{(k)} r_{i j}^{(k)}-\sum_{k=1}^{L} I_{i j}^{(k)} t_{i j}^{(k)}\right] .
$$

## Solution

- $I_{i j}^{(k)} r_{i j}^{(k)}$ is known to each receiver $j$, and so the sum (over $k$ ) is easily performed, and we can compute the sum over $j$ using a SDS as before
- the sum $\sum_{k=1}^{L} I_{i j}^{(k)} t_{i j}^{(k)}$ is a dot product, and so we use SDS to get two parts of this $s_{i j}^{(t)}$ and $s_{i j}^{(r)}$.
- $s_{i j}^{(t)}$ goes to the transmitter, and so we can perform a standard sum over $j$ on these
- $s_{i j}^{(r)}$ goes to the receivers, so we sum using a SDS
- $M_{i}$, the total number of received packets (transmitted from $i$ ) can be computed using a SDS
- transmitter gets all the info. to compute $\bar{D}_{i}^{\text {out }}$


## Oblivious transfer [?, ?]

- there are various versions
- consider 1-in-n Oblivious Transfer (OT)
- Alice has a list of numbers $\left\{a_{1}, a_{2}, \ldots, a_{n}\right\}$
- Bob has an index $\beta$
- Bob wants to learn $a_{\beta}$
- Alice must not learn $\beta$, and Bob must not learn $a_{i}$ for any $i \neq \beta$.
- Bob learns exactly one item from Alice's list, without Alice learning which item Bob discovered.


## Applications

- the millionaires problem
- more generically: calculating a minimum
- Assume Alice has wealth $w_{A} \in[1, n]$, and Bob has $w_{B} \in[1, n]$, where $n$ is known to both

```
Alice creates a 0
list of n numbers 0
w w
    If Bob gets 0
        then Bob is poorer
    If Bob gets 1
    then Bob is at least as rich
```

- implications for many game theory problems: prisoner's dilemma
- assumption that lies behind the dilemma: the prisoner's can't trust each other
- Secure distributed computing provides mechanisms for creating trust
- end up with more co-operations
- example: inter-ISP traffic engineering


## Conclusion

- we can do stuff that I never imagined (until very recently)
- some of it is really cool
(*) - no real millionaires were harmed in the production of these slides


## Bonus slides

## OT - how it works

1-in-2 Oblivious Transfer

- Alice has a pair of bits ( $a_{0}, a_{1}$ ), and Bob has $\beta$
- trapdoor permutation $f$
- Given key $k$, can choose permutation pair ( $f_{k}, f_{k}^{-1}$ )
- Given $f_{k}$ it is hard to find $f_{k}^{-1}$
- Easy to choose random element from $f_{k}$ 's domain
- random Bit $B_{f_{k}}$ is a poly.-time Boolean function
- $B_{f_{k}}=1$ for half of the objects in $f_{k}^{\prime}$ s domain $B_{f_{k}}=0$ for other half
- no probabilistic polynomial time algorithm can make a guess for $B_{f_{k}}(x)$ that is correct with probability better than $1 / 2+1 / \operatorname{poly}(k)$


## 1-in-2 Oblivious Transfer

- A randomly chooses $\left(f_{k}, f_{k}^{-1}\right)$, and tells $f_{k}$ to $B$
- $B$ randomly chooses $x_{0}$ and $x_{1}$ in $f_{k}^{\prime}$ 's domain, and computes $f_{k}\left(x_{i}\right)$
- $B$ sends $A$ the pair

$$
(u, v)= \begin{cases}\left(f_{k}\left(x_{0}\right), x_{1}\right), & \text { if } \beta=0 \\ \left(x_{0}, f_{k}\left(x_{1}\right)\right), & \text { if } \beta=1\end{cases}
$$

- A computes $\left(c_{0}, c_{1}\right)=\left(B_{f_{k}}\left(f_{k}^{-1}(u), f_{k}^{-1}(v)\right)\right)$
- $A$ sets $d_{i}=a_{i}$ xor $c_{i}$ and sends $\left(d_{0}, d_{1}\right)$ to $B$
- $B$ computes $a_{\beta}=d_{\beta}$ xor $B_{f_{k}}\left(x_{\beta}\right)$


## SDP - how it works

(1) A and B agree on two numbers $m$ and $n$
(2) A finds $m$ random vectors $\mathbf{t}_{i}$ such that

$$
\mathbf{a}_{1}+\mathbf{a}_{2}+\ldots+\mathbf{a}_{m}=\mathbf{a}
$$

B finds $m$ random numbers $r_{1}, r_{2}, \ldots, r_{m}$.
(3) for $i=1$ to $m$
(3a) A sends B $n$ different vectors:

$$
\left\{\mathbf{a}_{i}^{(1)}, \mathbf{a}_{i}^{(2)}, \ldots, \mathbf{a}_{i}^{(n)}\right\}
$$

where exactly one $\mathbf{a}_{i}^{(q)}=\mathbf{a}_{i}$, the other
$n-1$ vectors are random
(3b) B computes $\mathbf{a}_{i}^{(j)} \cdot \mathbf{b}-r_{i}$
(3c) A uses 1-in-n OT to retrieve

$$
v_{i}=\mathbf{a}_{i}^{(q)} \cdot \mathbf{b}-r_{i}=\mathbf{a}_{i} \cdot \mathbf{b}-r_{i} .
$$

(4) B computes $V_{b}=\sum_{i=1}^{m} r_{i}$
(5) A computes

$$
V_{a}=\sum_{i=1}^{m} v_{i}=\sum_{i=1}^{m} \mathbf{a}_{i} \cdot \mathbf{b}-r_{i}=\mathbf{a} \cdot \mathbf{b}-V_{b} .
$$

